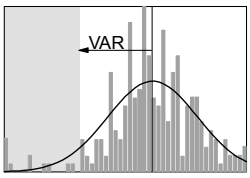
Managing Market Risk



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July 2004

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Evolution in Applications of VAR

Passive:
Reporting risk:
- disclosure to shareholders
- management reports
- regulatory requirements

Defensive:
Controlling risk:
- setting risk limits
  (desk level and firm-wide)

Active:
Allocating risk:
- performance evaluation
- capital allocation
- strategic business decisions

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Managing Market Risk: PLAN

(1) Managing risk with VAR tools: marginal VAR, incremental VAR, and component VAR

(2) From risk measurement to portfolio management

(3) Liquidity risk and horizon issues

(4) Systemic effects of VAR systems

(5) Pitfalls in VAR

Risk Management

(1)
Managing risk with VAR tools

Portfolio Risk

- Portfolio return: $P = w' R$ (w are weights)
- Portfolio variance: $\sigma_p^2 = w' \Sigma w$
- Portfolio VAR: $\text{VAR} = \alpha W \sigma_P$ (W is wealth)
- VAR and correlations:
  - » in general, $\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2\rho w_1\sigma_1 w_2\sigma_2$
  - diversified VAR: $\text{VAR}_p < \text{VAR}_1 + \text{VAR}_2$
  - » unit correlations $= w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1\sigma_1 w_2\sigma_2$
  - undiversified VAR: $\text{VAR}_p^{UND} = \text{VAR}_1 + \text{VAR}_2$

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VAR Tools: Incremental VAR

- We consider an addition a (new trade) to the initial portfolio p
  - » the new trade is described by a vector of exposures on risk factors
- Does the trade increase or decrease risk?
  - » this is a non-linear measure
- Incremental VAR: the difference in portfolio VAR with and without a given position "a"

$$\text{IVAR} = \text{VAR}_{p+a} - \text{VAR}_p$$

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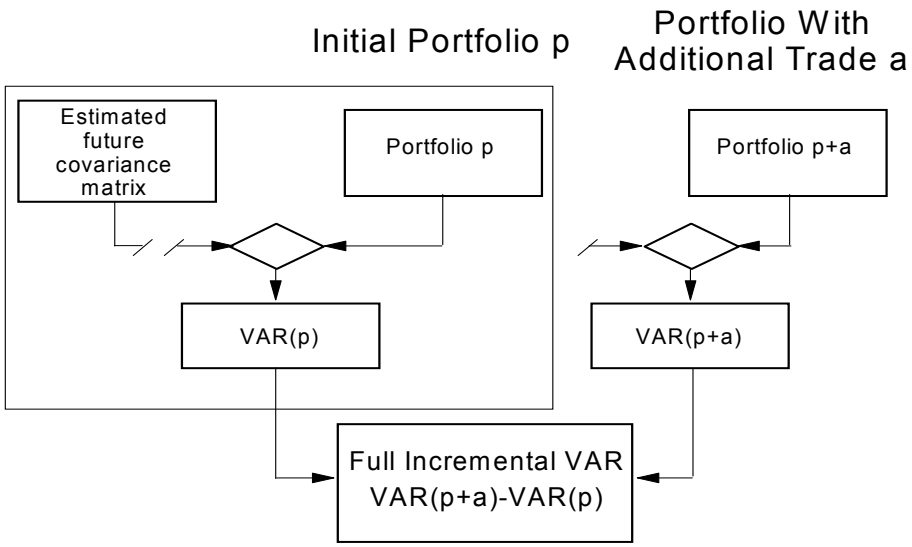
Incremental VAR



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VAR Tools: Marginal VAR

- We want to describe the linear change in VAR for a small change in each position
- This reveals which risk factor change can potentially reduce most VAR
- Marginal VAR: the change in portfolio VAR resulting from taking an additional unit of exposure to a given component

$$\Delta \text{VAR}_i = \partial \text{VAR}_p / \partial w_i = \partial(\alpha \sigma_p W)/\partial\, w_i$$

$$\boxed{\Delta \text{VAR}_i = \text{VAR} \times \beta_i}$$

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VAR Tools: Component VAR

- We want to decompose the total VAR into percentage attributable to each risk factor
- Component VAR: a partition of the portfolio VAR that indicates how much VAR would approximately change if the component was deleted

$$\text{VAR}_p = \text{CVAR}_1 + \text{CVAR}_2 + \ldots$$

$$\boxed{\text{CVAR}_i = \text{VAR} \times \beta_i \, w_i = \Delta\text{VAR}_i \, w_i}$$

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VAR Tools: Example

	Position	Volatility	Individual VAR	Marginal VAR	Component VAR
	x_i	σ_i	$\alpha \, x_i \, \sigma_i$	ΔVAR_i	$\Delta\text{VAR}_i \, x_i$
Can.Dol	$2,000,000	5%	$165,000	0.0528	$105,630
Euro	$1,000,000	12%	$198,000	0.1521	$152,108
Sum	$3,000,000				$257,738
		Undiversified VAR:	$363,000		
		Diversification effect:	$105,262		
		Diversified VAR:	$257,738		

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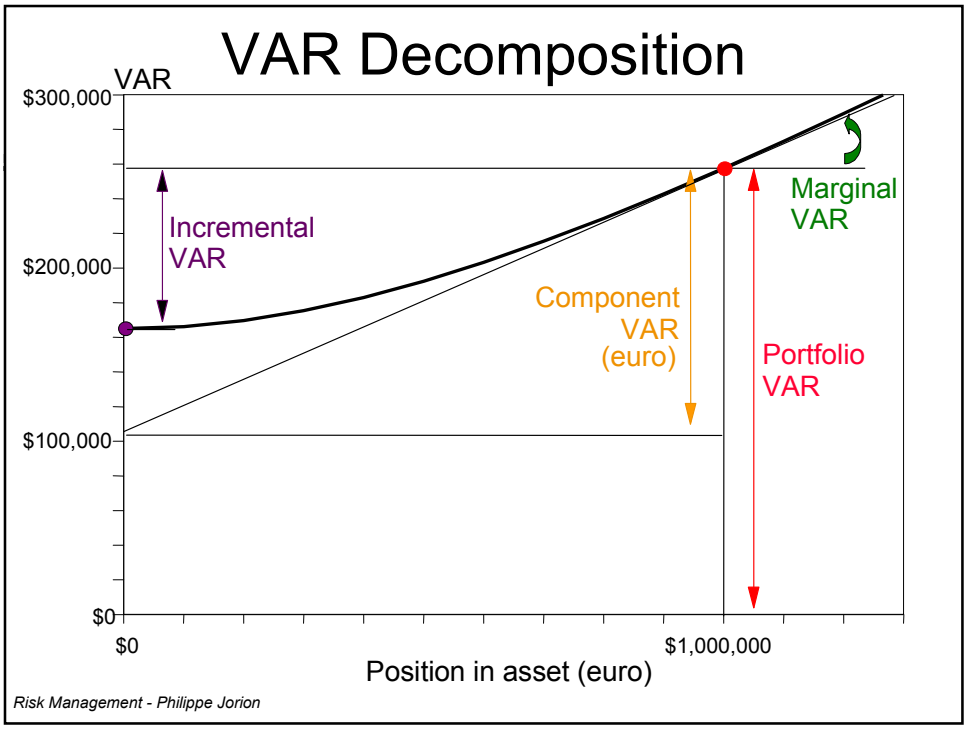

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Value-at-Risk of a Global Portfolio

Input Cells | Output Cells

INSTRUMENT	CODE	POSITION (DOLLARS)	Weight (%)	Volatility (%)	INDIVIDUAL $ AT RISK	MARGINAL $ AT RISK	COMPONENT $ AT RISK	Percent Contribution
Spot-DEM	DEM.XS	$0	0.0%	4.935	$0	$0.0073	$0.0	0.0%
Spot-JPY	JPY.XS	$0	0.0%	6.044	$0	($0.0240)	$0.0	0.0%
Euro90d-DEM	DEM.R090	$0	0.0%	0.050	$0	$0.0000	$0.0	0.0%
Euro90d-JPY	JPY.R090	$0	0.0%	0.042	$0	($0.0001)	$0.0	0.0%
Euro90d-USD	USD.R090	$0	0.0%	0.045	$0	$0.0002	$0.0	0.0%
Zero05-DEM	DEM.Z05	$0	0.0%	1.553	$0	$0.0118	$0.0	0.0%
Zero10-DEM	DEM.Z10	$0	0.0%	2.614	$0	$0.0186	$0.0	0.0%
Zero05-JPY	JPY.Z05	$0	0.0%	1.268	$0	$0.0025	$0.0	0.0%
Zero10-JPY	JPY.Z10	$0	0.0%	2.469	$0	$0.0059	$0.0	0.0%
Zero05-USD	USD.Z05	$0	0.0%	1.719	$0	$0.0272	$0.0	0.0%
Zero10-USD	USD.Z10	$10,000	100.0%	3.288	$543	$0.0543	$542.6	100.0%

			UNDIVERSIFIED:		
TOTAL	$10,000		$543	$542.6	100.00%
NET INVESTMENT:	**$10,000**	100.00%		516.4834354	
				$26.0796	

TOTAL DOLLARS AT RISK=
(95% PROBABILITY LOSS WON'T EXCEED THIS AMOUNT)

DIVERSIFIED VAR:
In 1 month **$ 543**
In 1 year $ 1,879

PORTFOLIO VARIANCE	1.08E-03
1 MONTH PORTFOLIO VOLATILITY (%)	3.288%
12 MONTH PORTFOLIO VOLATILITY (%)	11.391%

VAR Graph

Prof. P.Jorion

Value-at-Risk of a Global Portfolio

Input Cells | Output Cells

INSTRUMENT	CODE	POSITION (DOLLARS)	Weight (%)	Volatility (%)	INDIVIDUAL $ AT RISK	MARGINAL $ AT RISK	COMPONENT $ AT RISK	Percent Contribution
XS	DEM.XS	$3,000	17.6%	4.935	$244	$0.0595	$178.6	34.6%
XS	JPY.XS	$4,000	23.5%	6.044	$399	$0.0706	$282.4	54.7%
R090	DEM.R090	$3,000	17.6%	0.050	$2	$0.0001	$0.2	0.0%
R090	JPY.R090	$0	0.0%	0.042	$0	($0.0001)	$0.0	0.0%
R090	USD.R090	$0	0.0%	0.045	$0	$0.0000	$0.0	0.0%
Z05	DEM.Z05	$0	0.0%	1.553	$0	$0.0035	$0.0	0.0%
Z10	DEM.Z10	$0	0.0%	2.614	$0	$0.0027	$0.0	0.0%
Z05	JPY.Z05	$0	0.0%	1.268	$0	$0.0025	$0.0	0.0%
Z10	JPY.Z10	$4,000	23.5%	2.469	$163	$0.0050	$20.0	3.9%
Z05	USD.Z05	$0	0.0%	1.719	$0	$0.0076	$0.0	0.0%
Z10	USD.Z10	$3,000	17.6%	3.288	$163	$0.0118	$35.4	6.8%
				VAR:				
TOTAL		$17,000	100.00%	UNDIVERSIFIED	$971		$516.5	100.00%
Net Investment		$10,000		DIV.EFFECT:	$455			

TOTAL DOLLARS AT RISK=
(95% PROBABILITY LOSS WON'T EXCEED THIS AMOUNT)

DIVERSIFIED VAR:
In 1 month **$516.5**

PORTFOLIO VARIANCE	3.39E-04
1 MONTH PORTFOLIO VOLATILITY (%)	1.841%
12 MONTH PORTFOLIO VOLATILITY (%)	6.378%

VAR Graph

Risk Management

(2)

From risk measurement to portfolio management

From Risk Measurement to Risk Management

- Risk management is much more than passive risk measurement
- The ultimate goal is to move to the best risk-return profile for the portfolio
- In practice, risk measures must be combined with position taking, or expected returns
- The portfolio should move toward the "Mean-Variance efficient frontier", which provides the best combination of risk and return

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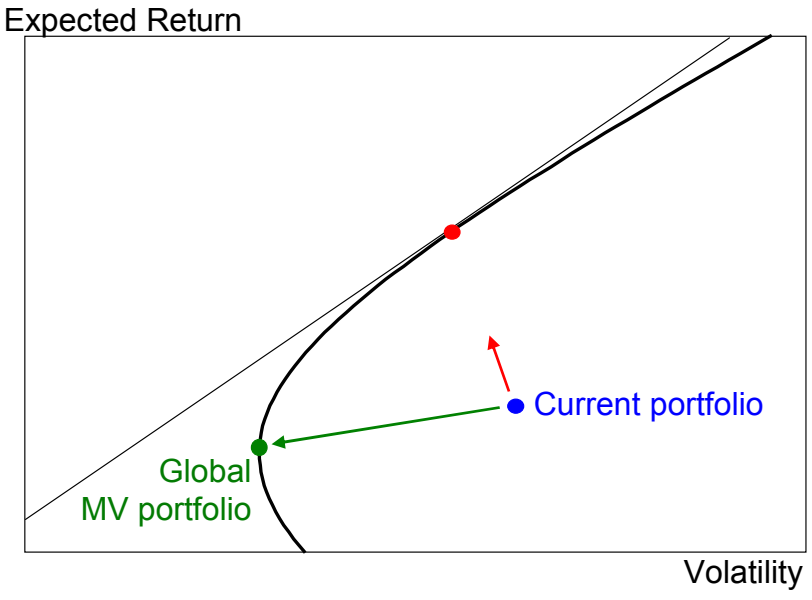
Risk-Return Trade-Off



From Risk Measurement to Optimization

- All MV efficient portfolios B must be such that expected returns on all assets satisfy:

$$E(R_i) - R_f = \beta_i [E(R_B) - R_f]$$

- To go from the current position to an efficient portfolio, sort all assets by the ratio of excess return to marginal VAR (or beta):

$$\text{T-ratio} = [E(R_i) - R_f] / [\text{VAR } \beta_i]$$

- Invest in asset with highest T-ratio
- At optimum, all T-ratios are equal

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Risk Management

(4)
Liquidity and Horizon

ASSUMPTIONS BEHIND VAR

- Take VAR = $\alpha \sqrt{(x_t' \Sigma_{t+1} x_t)}$
- This assumes a frozen portfolio over horizon
  - » ignores prudent risk management systems
  - » ignores options
- If we want to measure the worst mark-to-market loss, we can use VAR with mid-rates
- If we want to measure the worst loss under liquidation, we need to consider liquidation strategies and price pressure effects

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What is the Horizon?

- Usually fixed
  - » 1-day for internal market risk
  - » 10-day for regulatory market risk
  - » 1 year for credit risk
- Horizon should reflect the time required for corrective action: liquidation, hedging, raising capital
- Strategic response affects the risk profile
- Liquidation strategies should trade off price volatility against transaction costs

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Horizon and Liquidity Risk

- Liquidity risk is loosely accounted for by choosing a horizon which is longer than liquidation period
- Particularly illiquid assets can be factored in by increasing the volatility of these assets

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LIQUIDITY RISK

- Asset liquidity risk arises when a transaction cannot be conducted at prevailing market prices due to the size of the position relative to normal trading lots
- This involves:
  - » bid-ask spread, normal market size
  - » price-quantity function
  - » quantity in inventory

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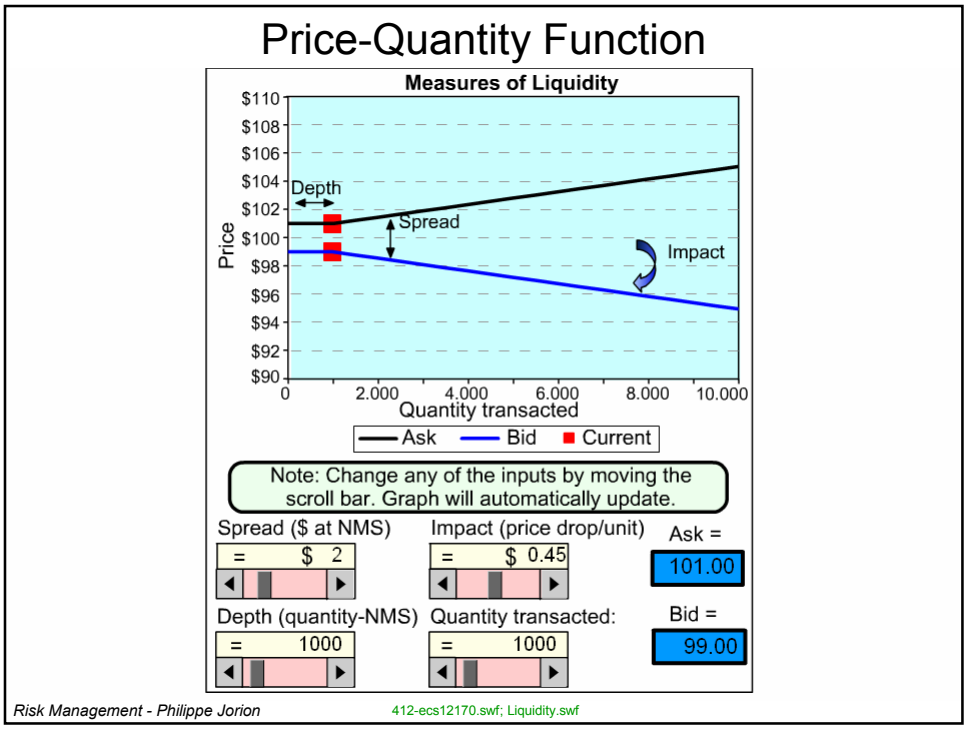
Price-Quantity Function
Measures of Liquidity
$110
$108
$106
$104
$102
$100
$98
$96
$94
$92
$90
Price
Depth
Spread
Impact
0
2.000
4.000
6.000
8.000
10.000
Quantity transacted
Ask
Bid
Current
Note: Change any of the inputs by moving the scroll bar. Graph will automatically update.
Spread ($ at NMS)
= $ 2
Impact (price drop/unit)
= $ 0.45
Ask =
101.00
Depth (quantity-NMS)
= 1000
Quantity transacted:
= 1000
Bid =
99.00
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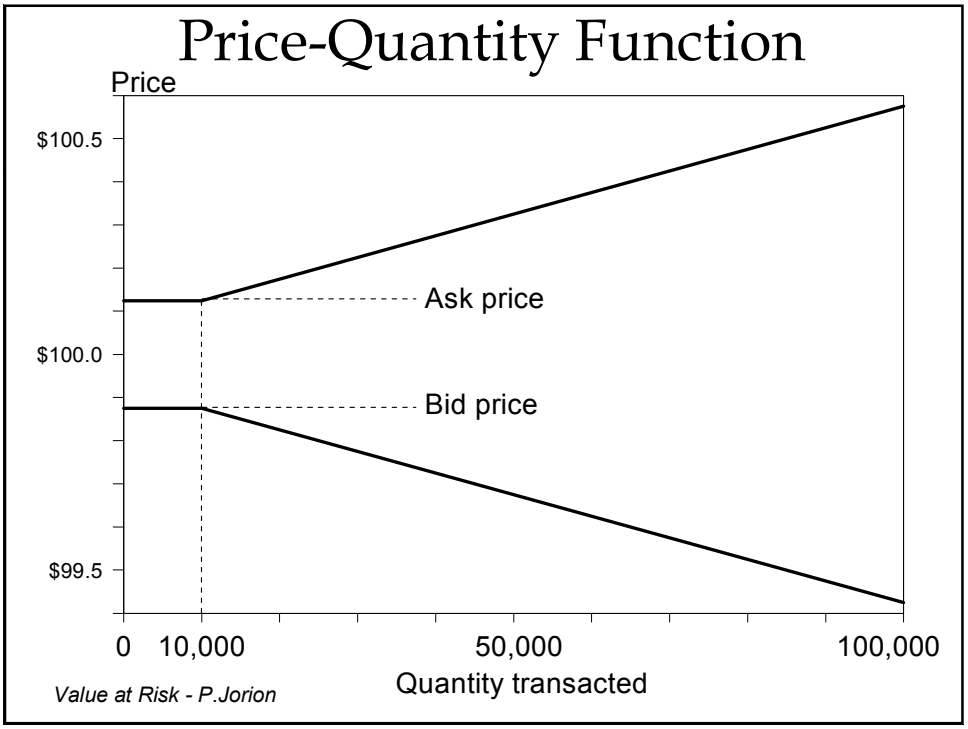
412-ecs12170.swf; Liquidity.swf


Price-Quantity Function
Price
$100.5
$100.0
$99.5
Ask price
Bid price
0
10,000
50,000
100,000
Quantity transacted
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Typical Liquidity Spreads

Typical Spreads and Volatility (Percent)			
Asset	Spread	Volatility	
	(bid-ask)	Daily	Annual
Currencies:			
major (Euro, yen, ...)	0.05-0.20%	0.3-1.0%	5-15%
emerging (floating)	0.50-1.00%	0.3-1.9%	5-30%
Bonds:			
on-the-run Treasuries	0.03%	0.0-0.7%	0-11%
off-the-run Treasuries	0.06-0.20%	0.0-0.7%	0-11%
corporates	0.10-1.00%	0.0-0.7%	0-11%
Treasury Bills:	0.003-0.02%	0.0-0.1%	0- 1%
Stocks:			
US	0.05-5.00%	1.3-3.8%	20-60%
average, NYSE	0.20%	1.00%	15%
average, all countries	0.40%	1.0-1.9%	15-30%

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LIQUIDITY-ADJUSTED VAR WITH EXOGENOUS SPREADS

- Liquidity-adjusted VAR:

$LVAR = VAR + L_1 = (\alpha W \sigma) + (1/2)(WS)$

e.g. $LVAR = (1.645 \times \$1m \times 1\%) + (1/2)(\$1m \times 0.25\%)$

$= \$16{,}450 + \$1{,}250 = \$17{,}700$

» effects typically second-order

- With many assets, add-ons $(1/2)\Sigma_i |W_i| S_i$, which increase at linear rate (unlike VAR)
- With uncertainty in spreads (μ_S, σ_S):

$LVAR = VAR + L_2 = (\alpha W \sigma) + (1/2)[W(\mu_S + \alpha' \sigma_S)]$

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LIQUIDITY-ADJUSTED VAR WITH LIQUIDATION STRATEGY

- If position is large, we need to account for price impact
- Define *n* as number of days to liquidation
- Assume linear function $P(q)=P_0(1-kq)$
  - » *e.g.* $P_0=\$100$, $k=0.5\ 10^{-7}$
  - » sell 1m shares in one day, drop=$5, total=$5m
  - » sell 200,000 over 5 days, drop=$1, total=$1m
- Liquidation strategies need to account for trade-off between costs and price volatility

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Liquidation Strategies: Simplistic

(1) Immediate liquidation (sell *q* now) has quadratic costs: $C_1=q\times[P_0-P_0(1-kq)]= kq^2P_0$
no price risk: $V_1=0$

(2) Uniform liquidation (sell *q/n* a day) has lower costs: $C_2=q\times[P_0-P_0 k(q/n)]= k(q^2/n)P_0$
price volatility:

$V_2 = \sigma^2q^2\times[(1-1/n)^2+(1-2/n)^2+\ldots+(1-(n-1)/n)^2]$

$V_2 = \sigma^2q^2\times[([(n/3)(1-1/n)(1-1/2n)] = \sigma^2q^2\ T^*$

» e.g. n=5 days, T^*=1.2; Basel T^*=10, n=31

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Liquidation Strategies: Optimal

(3) Optimal tradeoff: find the trajectory *{x}* that

$$Min_x [C_x + \lambda V_x]$$

- Optimal position of the type:

$$x(t) = q \times \sinh[\kappa(T-t)]/\sinh[\kappa T]$$

- Compute liquidity-adjusted VAR

$$LVAR = \alpha\sqrt{V(W)} + C(W)$$

  - » solutions with linear costs in Almgren&Chriss (2001), "Optimal execution of portfolio transactions," Journ.of Risk
  - » extensions by Konishiy and Makimoto (2001), "Optimal slice of a block trade," JOR

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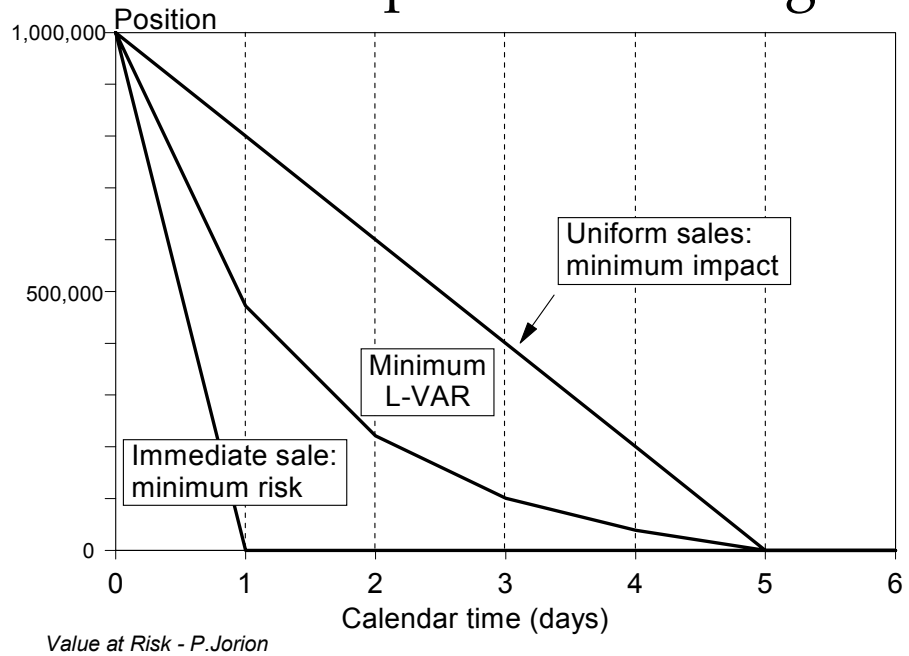
Profile of Liquidation Strategies



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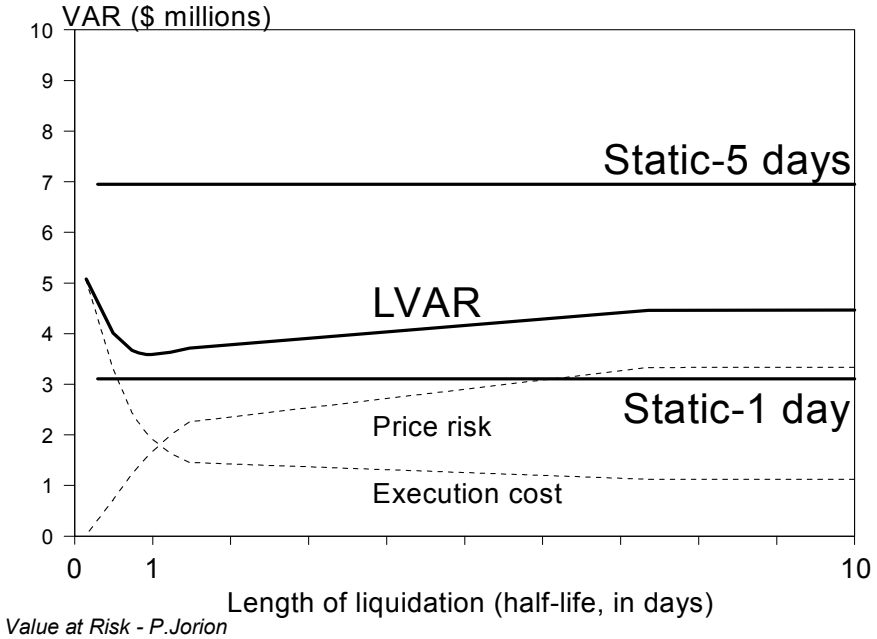
Liquidity-Adjusted VAR



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Liquidation Strategies: Example

- Portfolio of $100m with daily volatility of 1.89%
- 1-day VAR is $1.645 \times 1.9\% \times \$100m = \$3.1m$
- 2-day VAR is $\$3.1 \times \sqrt{2} = \$4.4m$
- Liquidation after 0 day (within 1day): cost=$5m
- Optimal liquidation after 1 day (within 2 days):
  - » optimal κ=0.92, positions=1,000,000; 345,000; 0
  - » cost= $0.5\ 10^{-7} \times (655{,}000^2 + 345{,}000^2) = \$2{,}148{,}000$
  - » risk= $\$1.9 \times 345{,}000 = \$650{,}000$
  - » LVAR= $1.65 \times \$650{,}000 + \$2{,}148{,}000 = \$3.8m$

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LIQUIDITY AND MANAGEMENT INTERVENTION

- Illiquidity increases worst loss (in liquidation scenario)
- Limit management decreases worst loss
- Basel rules requires capital to cover

$$\text{CAPITAL} = m \times 3\sqrt{10}\ \text{VAR}_{99\%}$$

- Consider a leveraged position with dynamic capital; default occurs if capital falls below 0
- Dynamic management accounts for:
  - » liquidity (longest period T to liquidate position)
  - » management intervention (period R between resetting limits)

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Dynamic Capital

- Consider a leveraged position in S&P index
  - » start with $1 in capital
  - » set VARLimit so that: $\text{CAPITAL} = m3\sqrt{10}\ \text{VARLimit}$
  - » from this, position is: $\text{VALUE} \times \text{VAR}_{99\%} = \text{VARLimit}$
  - » position is funded as: $\text{VALUE} = \text{CAPITAL} + \text{DEBT}$
  - » compute P&L the next day: $P\&L = V \times R_{S\&P} - D \times i$
  - » add P&L to find new capital and VARLimit
  - » adjust the position every R days, accounting for liquidation over T days

    Sale=MIN(desired, maximum=VALUE/T)

Marrison, Schuermann and Stoughair, "Changing regulatory capital to include liquidity and management intervention," Journal of Risk Finance (Summer 2000)

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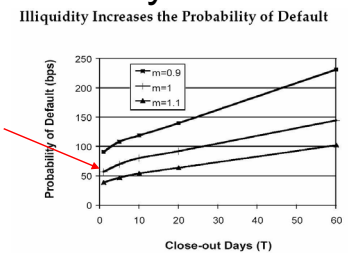
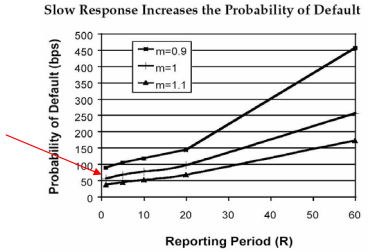
Dynamic Capital (2)

- With m=1, T=1, R=1, probability of default over the next year is P=0.57%





- Choose m to maintain P=0.84% (below BIS 1%)

m

Closeout Days (T)

Reporting Period (R)	1	5	10	21	62	∞
1	0.92	0.96	0.99	1.02	1.11	**1.79**
5	0.95	0.98	1	1.04	1.12	**1.79**
10	0.98	1.00	1.03	1.06	1.17	**1.79**
21	1.04	1.05	1.07	1.09	**1.21**	**1.79**
62	**1.21**	**1.21**	**1.21**	**1.21**	**1.38**	**1.79**
250	**1.79**	**1.79**	**1.79**	**1.79**	**1.79**	**1.79**

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Risk Management

(5)
Pitfalls in Risk Management

LIMITATIONS OF VAR (1-2)

- Risk of Exceedences:
  - » VAR does not measure the worst loss
  - » it will be exceeded on a regular basis (1–c)
- Changing Positions Risks:
  - » VAR assumes fixed positions over horizon
  - » adjustment ignores that the trading position might change over time in response to changing market conditions
  - » no easy solution, but “prudent”' dynamic trading patterns will decrease risk relative to conventional VAR measures

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LIMITATIONS OF VAR (3-4)

- Event and Stability Risks:
  - » historical data subject to one-time events or structural changes
  - » use stress-testing, or models that allow risk to change through time, or implied vols
- Transition Risk:
  - » whenever there is a major change, a potential exists for errors
  - » e.g. organizational changes, expansion into new markets or products, implementation of a new system, or new regulations

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LIMITATIONS OF VAR (5)

- Data-Inadequacy Risks:
  - » insufficient data to evaluate risks and correlations:
  - (i) infrequently traded emerging market stocks
  - (ii) private placements
  - (iii) exotic currencies

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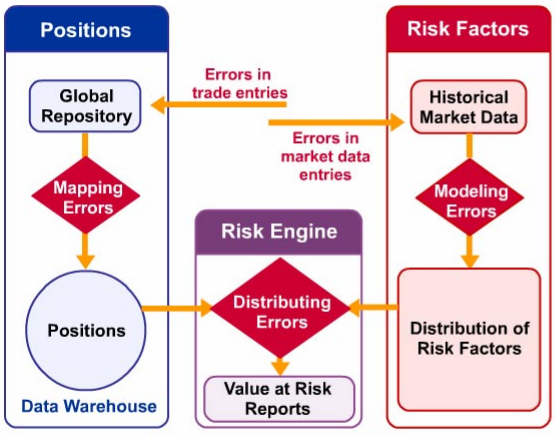
Model Risk

Any measurement system is subject to **model risk**, which is the risk of losses due to inappropriate models or data. Model risk is a form of **operational risk**, that is, the risk that processes, systems and people can fail, leading to undesirable effects on a business.



Errors can occur at any point during the risk measurement process:

Continue

- When trades and market data are entered into the system
- During the mapping process
- When risk factors are statistically modeled
- When computing the distribution of profits and losses

Once a risk measurement system is in place, traders may have an incentive to find trades that have low reported risk, and exploit holes in the VAR system. The potential for such **VAR arbitrage** must be constantly monitored by risk managers. In the end, experienced risk managers are more important than sophisticated systems.

VAR arbitrage: The deliberate creation of risky trades that appear to have low risk in a VAR framework.

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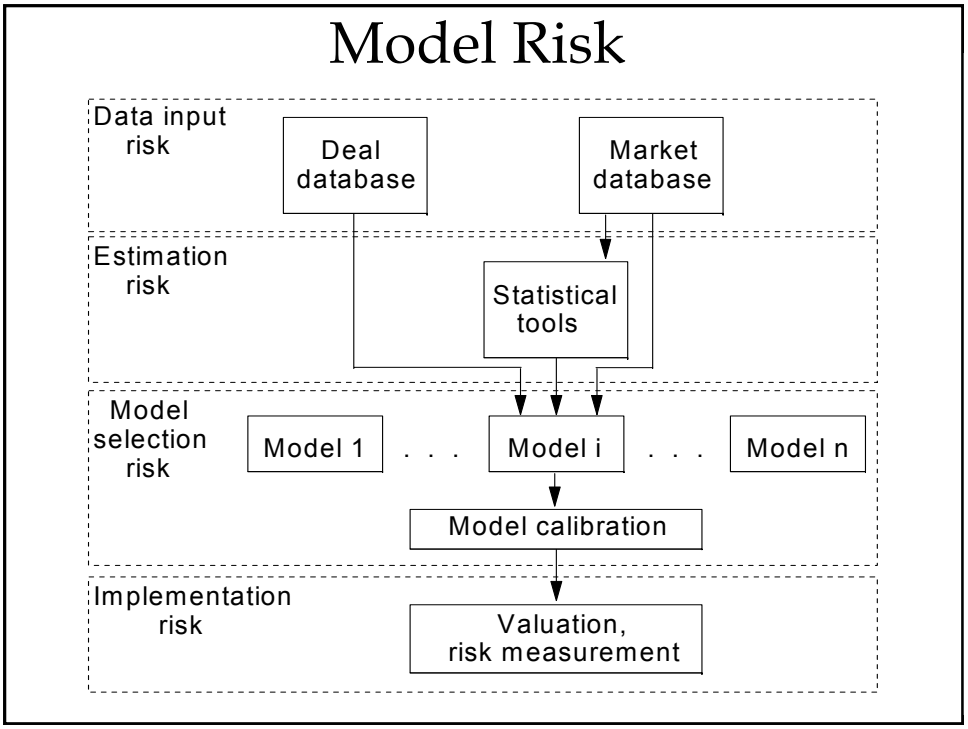

LIMITATIONS OF VAR (6a): Model Risk

(1) Data Input Risk:
- » deal and market data may have mistakes

(2) Estimation Risk:
- » imprecision in the measurement of parameters
- » survival of series

(3) Model-Selection Risk:
- » valuation errors if pricing model is incorrect
- » data mining when searching over various models

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LIMITATIONS OF VAR (6b): Model Risk

(4) Implementation Risk:
- » different numerical methods, mapping
- » bugs in software
- » user risk even with the same software
- » model risk larger for more complex instruments

	Forwards	Money Market	FRAs	Global Bonds	Int.Rate Swaps	FX Options	Int.Rate Options
Notional	$130m	$46m	$375m	$350m	$311m	$374m	$327m
1-d 95% VAR:							
Median	$425,800	$671,300	$79,000	$3,809,100	$311,100	$804,200	$416,700
Std.Dev.	$4,800	$60,700	$7,500	$652,800	$66,600	$198,800	$115,200
Ratio	1%	9%	10%	17%	21%	25%	28%

Source: Marshall and Siegel (1997)

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SIDE EFFECTS OF VAR (1-2)

(1) "Man in White Coat" syndrome:
- » VAR systems give false sense of security, hence are dangerous
- » however, financial engineering is an art form

(2) Traders gaming system: "VAR arbitrage", or the deliberate creation of risky trades that appear to be low risk in a VAR framework
- » pegged currencies with zero historical volatility
- » HS methods with short windows
- » analytical methods invite option trades

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SIDE EFFECTS OF VAR (3-4)

(3) Portfolio optimization:
- » a systematic form of VAR arbitrage that amplifies estimation error in risk and correlation measures
- » problem is real

(4) Dynamic hedging:
- » VAR constraints could create automatic trading, increasing volatility?
- » however, anecdotal evidence

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Measuring Market Risk

Conclusions

CONCLUSIONS (1)

- The ultimate goal of risk measurement is to understand risk better so as to manage it effectively
- Risk management should not only prevent losses, but add value to the decision process: it supports portfolio management
- Tools such as marginal and component VAR are integral to portfolio management, because they allow optimal tradeoff between risk and return

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CONCLUSIONS (2)

- VAR methods involve many assumptions:
  - » frozen portfolio
  - » risk measures extrapolated from historical data, typically with zero correlations
  - » risk at horizon only
- Realistically, risk measures should reflect
  - » changing risk profile
  - » optimal liquidation strategies
- Measuring VAR should lead to better risk management

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CONCLUSIONS (3)

- It is important to be aware of limitations of VAR methods, such as “model risk”
- Worst danger is “VAR arbitrage”, where traders exploits risk holes in system
- Risk managers need to understand risks of trading strategies; stress testing is important
- Effective risk management requires competent risk managers

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References

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